A Mathematical Approach to Psychology

George A. Kelly

The Ohio State University

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Many different approaches to psychology are possible, but most of them have not been invented yet. The purpose of this paper is to describe an approach that is now in the process of being invented. Like all the others which have preceded it, as well as those to come later, it is a construction and not a discovery. I must make this clear at the outset; I did not find this theory lurking among the data of an experiment nor was it disclosed to me on a mountain top or in a laboratory, I have, in my own clumsy way, been making it up.

INVENTION AND DISCOVERY IN SCIENCE

A scientist's inventions assist him in two ways: They tell him what to expect and they help him see it when it happens. Those that tell him what to expect are theoretical inventions and those that enable him to observe outcomes are instrumental inventions. The two types are never wholly independent of each other, and they usually stem from the same assumptions. This is unavoidable. Moreover, without his inventions, both theoretical and instrumental, man would be both disoriented and blind. He would not know where to look or how to see.

While invention is the key to progress in science, as well as in other forms of human inquiry, discovery is important also. But when we speak of discovery we must be explicit about what we mean, else we shall find ourselves enmeshed in the same difficulties as those who think that the natural events of the world go around introducing themselves by name and whispering theoretical revelations into the ears of deserving scientists.

First of all, theoretical inventions are used to make predictions. Then, still using inventions—but of a more instrumental type, we examine the outcomes to see if there is any correspondence between what we have predicted and what our instrumentalized perceptions tell us has occurred. If we find such a correspondence, we call it a discovery. We do not discover our theory; we do not discover our prediction; we do not even discover the ensuing event. What is discovered is a correspondence—a practical correspondence—between what our theoretical invention leads us to anticipate and what, subsequently, our instrumental invention leads us to observe.

When a scientist repeatedly fails to find any correspondence between what appears to happen and what his theoretical invention has led him to expect, he is likely to conclude, sooner or later, that his theory is worthless. Thus, if the invention I am about to describe to you fails to produce expectations that materialize, I, too, am likely to conclude, sooner or later, that my invention is worthless. Others may reach this conclusion sooner than I, for, having gone to the trouble of inventing the theory and writing papers about it, I shall want to explore its possibilities extensively before abandoning it. Some of you may be happy to abandon the theory at the very outset, while others may wish to pursue it, along with me, for some distance, if only out of sheer curiosity to see where it leads.

This paper, then, begins with a personal invitation to you to join me aboard my theoretical vessel and set out on a voyage of discovery, and, sharing with me such instruments as I have on board, to observe the islands we pass. If, at any time, you despair of discovering any correspondence between what we seek and what looms up on the horizon, you are free to turn back. I, too, may turn back at some point, if I become discouraged with the outcomes of our venture, while some of you, who are not so easily discouraged, may press the voyage further.

CONSTRUCTIVE ALTERNATIVISM

These statements I have been making so far are more epistemological than psychological. But let me continue further with them, for they have a bearing on the psychological statements I shall want to make later.

For a long time scientists have assumed that before they could advance a new theory they must first prove something wrong with the old ones. I do not consider this a

necessary assumption. It is true that our disappointments with one kind of explanation do often serve to set us off in search of a better one. There is nothing wrong with that. But does one always have to wait until he is frustrated and embittered before he dares start looking for new horizons? I think not.

The adventure in which I have invited you to join me for a little while does not, therefore, require you to deny anything you now believe or to destroy anything you now find useful. That is why I have said that you are free to return whenever you find the voyage discouragingly unproductive. You need not scuttle your present ships in order to embark on this one. Nor need you wait until you are discouraged before you quit my vessel for another.

The question of whether a theory is true or false, good or bad, useful or futile is not identical with the question of whether or not to explore its implications. These two types of questions arise properly at different times, and therefore are not to be answered concurrently. This is a point of view which is a convenience in dealing with what I have to offer; but it is more than that—it is an essential feature of the theory itself, as I hope to show during the course of this paper.

The underlying philosophical position which I have sketched briefly in the preceding paragraphs may be called, "constructive alternativism," This is to say that reality is subject to many alternative constructions, some of which may prove to be more fruitful than others. The discovery of an ultimate correspondence between the constructions we are able to devise and the flow of actual events is an infinitely long way off. In the meantime, we shall have to be content to make a little progress at a time, to invent new alternative constructions—even before we have become dissatisfied with the old ones, and hope that, in general, we are moving in the right direction.

CONVERGING LINES OF INFERENCE: I. GENERALIZATION OF THE PSYCHOLOGY OF SCIENTIFIC BEHAVIOR

Now, having said something philosophical, we are prepared to go on to say something more psychological. Since, as I am sure Professor Luria will agree, it is helpful to have a verbal response for something if we are to control all our other reactions to it, let me say that the theory I am about to describe may be called, "the psychology of personal constructs."

The theory may be said to represent the convergence of several lines of inference. Naturally, I shall not attempt to describe all of them, but it is appropriate to mention one or two.

One thing that has struck me is that nearly every psychologist of our time inadvertently uses two quite different systems for explaining human behavior. While I see nothing particularly wrong in this, as you already know from my discussion of constructive alternativism, it does raise the fascinating question of whether one or the other of the two systems might not suffice to explain the whole spectrum of human behavior.

Let me try to explain what I mean. Most psychologists consider themselves scientists. This is true in my country as well as in yours. They see science as progressing according to certain principles and by means of certain methodologies. But when these same psychologists turn their attention to man, they speak in another language. In the first chapter of their books they say that science progresses by inducing or deducing theoretical statements, by formulation of hypotheses, by specifying predictions, by experimentation, by observing outcomes and comparing them with expectations, and by constant revision of one's line of reasoning. But in the ensuing chapters they forget all about this and they attempt to describe man's behavior in quite different terms, the particular terms depending somewhat upon their theoretical orientation at the moment.

But science is itself a form of human behavior, and a pretty important one, at that. Why, then, should we feel compelled to use one set of parameters when we describe man-the-scientist and another set when we describe man-the-laboratory-subject? I pose this question, not as one to be answered immediately by logical inference, or to be dismissed with the supposition that scientific behavior must require a unique psychology because it constitutes so small a part of human behavior, but as a question to be explored. I pose it, not as a philosophical question, but as a psychological one—and hence one to be answered or reformulated by scientific inquiry.

Answering a question of this sort, as I am sure you will agree, involves a good deal more than one simple laboratory experiment. One way of exploring this question is to take our notions of scientific progress—philosophical notions of what science is

and how it proceeds, remove them from the context of speculative philosophy and logic, and elaborate them as a system of psychological theory of man's behavior. Thus we would prepare our question for scientific investigation. We could then deduce hypotheses, raise issues, develop methodologies, devise instruments, generate data, perform experiments, induce further hypotheses, and revise our theoretical formulations, The psychology of personal constructs is an attempt to prepare our question in just such a way,

CONVERGING LINES OF INFERENCE: II. THE DOUBLE ENTITY CHOICE

Let me mention another of the converging lines of inference—merely one of many—which points to the psychology of personal constructs. From the time of Aristotle we have understood prepositional speech as a way of denoting entities. We may say, "A is B." This statement is a way of asserting a conclusion the antithesis of which would have been to say, "A is not B." This is a familiar logical form and its intrinsic validity is generally taken for granted.

But we can look at this matter as psychologists and pose a scientific question about it. When one says that "A is B," is he, in fact, merely abandoning the alternative proposition that "A is not B?" Or is he, in fact, denying some other alternative? This is a question about human behavior, not a question of classical logic. Since it is a question about human behavior, it should be open to psychological examination.

Experience with clients undergoing psychotherapy, as well as with persons in the process of changing their lives under other conditions, leads one to suspect that a person never makes his choice merely between an entity and a non-entity. When he says that "A is B" it seems that he is also asserting that "A is not C." The choice he makes is not, therefore, between "B" and "not-B," but between "B" and "C"—between two entities. Let us call this "the double entity choice," to distinguish it from "the single entity choice" envisioned by classical logic.

If you prefer, we can state this observation in behavioral terms. ¥e may then say that a person never chooses between behaving in a certain way and not behaving at all. Rather, he chooses between one behavior and another, He does not choose between activity and inactivity; instead, he chooses between alternative kinds of activity. At least, for the moment, this is the way it seems.

It must be clear that what I am offering here is an incident observation only. But it is an observation that can be pursued psychologically to see whether, as a generalization, it can be supported in a scientific manner. In order to examine it psychologically it is best that we first elaborate it in some theoretical form, preferably in one that is simple, coherent, communicable, productive of reasonably explicit hypotheses, and amenable to the operational definition of experimental terms,

And so, this time by a different route, we come again to the psychology of personal constructs, a theory which, among other things, attempts to do just this. It takes our incidental observation of the double entity choice in human behavior and incorporates it in a theoretical structure. The constructed theory, because of all its predictive implications, then becomes a basis upon which a series of scientific inquiries can be undertaken. Eventually, this series of inquiries should reveal to us whether or not we have invented anything useful.

THEORETICAL COMPOSITION OF THE PSYCHOLOGY OF PERSONAL CONSTRUCTS

Thus far in this paper we have mentioned only three main ideas. The first is constructive alternativism, a philosophical position which simultaneously sustains a vast variety of competing theoretical formulations, and the research that stems from them, even though some of those formulations appear as alternatives to what is currently acceptable. The other two ideas are the two incidental observations that give rise to converging lines of inference. The first of these suggests the possibility of generalizing the psychology of scientific behavior to all human behavior, and the second suggests the possibility of incorporating the double entity choice into a psychological theory. There are, of course, many other initial ideas and observations that might have been mentioned as leading to this theory, but these three are sufficient for this paper. Actually most of our further discussion will center around the third idea, the idea of the double entity choice, since this is the one that leads us most directly to an unusual kind of mathematical approach to psychology.

Suppose a child distinguishes between two objects, say, a ball and a cube. On the following day let us say he distinguishes between another ball and another cube. Why are his two performances similar? One thing we can say is that because the pairs of objects were physically similar on the successive days the child's responses

were similar. This explanation is based on the assumption that the child is under the control of the objects and therefore similar pairs of objects must always elicit similar responses. This is one type of explanation, and it is satisfactory, as far as it goes.

But suppose we make use of the principle of constructive alternativism and seek other kinds of explanation. Suppose we turn our attention to the child, rather than to the objects, and ask how it is that he was able to do with them what he did. Suppose we say it is not enough that the objects be similar; what more is required is that the child have some capability that enables him to respond as if they were similar. The psychological point I want to emphasize is that he construed the objects similarly, in spite of the fact that the occasions were different and the particular concrete objects were different.

One way of throwing light on the child's behavior is to examine the history of his performances. Such an examination has the advantage of enabling us to apply theories of conditioning or other historical types of explanation directly to particular acts. But, without burdening myself with the task of finding fault with such explanations—or others with the task of defending them, let me simply continue to take advantage of constructive alternativism and ask how the child, conditioned or not, copes with balls and cubes when they are presented to him.

One thing is obvious; the psychological feature we are seeking is itself neither a ball, a cube, a day, nor a set of circumstances—we are not looking for stimuli. The feature must be, instead, the child's own way of dealing with balls and cubes, his own way of channelizing his response to them, regardless of changing circumstances, and, therefore, one that he need not abandon at the end of the day when he goes to bed. Moreover, it is based on his personal construction of balls and cubes, a construction which he did not discover but which, as a kind of scientist, he invented. Since he is a "scientist" who is alive and active we may presume he is experimenting with his invention and is in the process of discovering its predictive utility.

It is at this point in the elaboration of the theory that I must pause to make a special stipulation. Nothing that I have said implies that the child's way of managing balls and cubes is necessarily based upon language or upon so-called conscious thinking. Nor do I mean to suggest that there are classical ideas or concepts floating around in his head. I have done no more than to invite your attention first to what the child

actually does, and now I am asking you to go one step further with me and pay particular attention to his ways of doing it. Thus, from now on, I shall be referring to the forms his life processes take, rather than to concrete processes themselves. The question of whether or not the processes are naturally physiological, mental, cognitive, spiritual, verbal, or unconscious is not relevant to what I have to say. We shall be talking about the ways these processes operate, not about their essences.

ASSESSING THE PERSONAL CONSTRUCT

Now that we have left behind the particulars of human behavior and are dealing with the abstractions of human behavior which we hope will provide the grounds for systematic scientific inquiry, we can start to delineate the forms into which we may cast those abstractions. We shall take our cue, as I have already indicated, from the observation that persona appear to make a double entity choice, rather than a single entity choice. Thus a child, in identifying a ball, appears to distinguish it from some other type of object—perhaps a cube—rather than merely picking it out, all at once, from all the things in the world which are not balls. If it appears to us that he picks it out all at once we may suppose that this is only because he has applied a sequential series of distinctions, and the types of objects he has eliminated along the way are not easily recognizable in his final identification of the ball.

If we give the child a ball, a cube, and a disk, and then ask him to put together the two that are alike, he can respond in at least three ways. (1) He can put the disk and the ball together, in which case we suspect that he has erected some construction that distinguishes a curvilinearity he ascribes to those two objects from an angularity. (2) He can put the cube and the disk together, in which case we suspect that the underlying distinction is between flatness and convexity. (3) Or he can put the cube and the ball together, in which case we might guess that he is distinguishing between thickness and thinness.

But how can we be sure? Suppose he actually puts the cube and the disk together, thus distinguishing them from the ball. Can we be sure that this means he has construed in the form, "convex versus flat?" Unless we are confident we can rely upon an exchange of language symbols with him, we must resort to the further explication of his construct by other means.

Suppose we next give him a lozenge and a feather. Suppose, instead of placing the convex lozenge beside the ball and the flat feather beside the cube and the disk, he does the reverse. Now he has the feather and the ball together, and on the other side he has the lozenge, the cube, and the disk.

Now that his construct has been explicated through five objects we may have a somewhat better understanding of the pattern his behavior follows. But we may still find it hard to predict accurately how he will arrange additional objects. We may hypothesize, at this point, that we can predict his further arrangements by using ourselves a construct of stable objects, such as the cube, the disk, and the lozenge, versus mobile objects which are easily dislodged, such as the ball and the feather* But this is only our hypothesis. Still, if by using it we do accurately predict his sorting of the next twenty objects given him, we may begin to feel some confidence that we have devised a useful notion of how his construct enables him to function in this simple laboratory situation.

THE PERSONAL CONSTRUCT AS AN ABSTRACTION OF HUMAN BEHAVIOR

When a person identifies an object we may say he has applied a construct to it. This is to say he has abstracted his behavior into a form we call the personal construct, and that he can now move consistently from situation to situation by the generalized application of this form. Indeed, when a person behaves discriminatingly it is quite likely that he has made use of several personal constructs. His employment of several constructs enables him to fix both objects and behaviors multidimensionally, as one fixes points geometrically in hyperspace.

The application of a construct to an object has certain implications. Let me put two of these implications systematically, so they may become an explicit part of the theory we are developing. The first implication is that there is at least one other imaginable object which stands in contrast to the one immediately construed. The second implication is that there is at least a third object which is similar to one of the other two.

You will recognize the first of these two systematic statements as a theoretical formulation based on the incidental observation I mentioned earlier—that persons make double entity choices. The second statement is required if we are to deal with the abstraction of behavior, rather than isolated incidents of behavior. We may go on

to say, then, that the minimum context in which a construct can be said to exist is three objects, or, to be more precise, three incidents. No less than three is required. As a matter of fact, of course, personal constructs are usually employed in much larger contexts.

It must also be understood that the personal construct abstracts similarity and difference simultaneously. One cannot be abstracted without implying the other. For a person to treat two incidents as different is to imply that one of them appears to be like another he knows. Conversely, for a person to treat two incidents as similar is to imply that he contrasts both of them with at least one other incident he knows. We intend this to be considered as an essential feature of the personal construct by means of which we hope to understand the psychology of human behavior.

THE PERSONAL CONSTRUCT AS A MATHEMATICAL FUNCTION

We come now to a more difficult point. I must confess that I find this point hard to explain to my students and colleagues. The discussion usually starts with the innocent question, "Do you envision the personal construct as a dichotomy or as a continuum?" My answer is, "I envision it as a dichotomy." But when I give this answer trouble starts. It appears to my listeners that I have said that human behavior must conform only to stereotypes and that everything in the world is judged as either black or white—never in shades of gray. This, of course, is not true.

I think I am beginning to understand what the root of the difficulty is. Most of us think about psychological matters concretely rather than abstractly. When we think of the form of human behavior we think of reflexes, of material learned, of decisions made, much as the child who, when he thinks of the mathematical value "four," thinks of "four apples," "four pieces of candy," "four pencils," or "four wheels."

Perhaps if we step outside the field of psychology for a moment we can make sure we have recovered our ability to think abstractly. Let us step into the field of geography. Consider the geographical construct of "north versus south." This is a dichotomous construct, and it is abstract. As far as the construct itself is concerned, there are no "partial norths" or "partial souths" crammed in between "north" and "south." And there certainly are no objects which are, of themselves, "north" or "south."

However, it is a simple matter to use this dichotomous construct to create an array of objects ranged from north to south. All we have to do is to take advantage of the fact that the construct is abstract, and therefore readily available for use in a wide variety of circumstances. We may then apply it sequentially to the different objects we want to place in the array. But the array of objects we have thus set in order is not the construct; it is only one kind of concrete explication of the construct.

We can go further. We can use our abstract construct to build a scale, as, for example, a scale of degrees along a meridian. This is simply a matter of creating an array of symbols which have been differentiated by our construct. Such a device has the advantage of being somewhat portable—and it is undoubtedly convenient. But never should it be confounded with the abstract construct of "north versus south," which is the basis for the device, and without which such a scale could never have been imagined.

Let us turn back, now, to psychology and visualize this same kind of dichotomous abstraction taking place. Let us visualize it taking place, not only with respect to geography, but with respect to all matters with which men, consciously or unconsciously, must cope. The particular behavioral content may vary from subject to subject and from person to person, but we propose the term, "personal construct" for the general form in which construing takes place.

As for the question of whether men deal with their world in terms of categories or continua, that is a heuristic matter. The fact is they do both, as we all well know. But the baseline, from which we may proceed to erect either categories or continua, and upon which we are free to project any behavior in our effort to understand it, may be regarded as essentially a dichotomous differentiating and integrating unit—the personal construct. It is in this sense, and in this sense only, that it properly becomes a mathematical function.

THE GEOMETRY OF PSYCHOLOGICAL SPACE

Now that we have our basic unit, the personal construct, partially defined, we can turn our attention to the question of how the world appears when structured in such terms. Perhaps it is already clear that our psychological geometry is a geometry of dichotomies rather than the geometry of areas envisioned by the classical logic of concepts, or the geometry of lines envisioned by classical mathematical geometries.

Each of our dichotomies has both a differentiating and an integrating function. That is to say it is the generalized form of the differentiating and integrating act by which man intervenes in his world. By such an act he interposes a difference between incidents—incidents that would otherwise be imperceptible to him because they are infinitely homogeneous. But also, by such an intervening act, he ascribes integrity to incidents that are otherwise imperceptible because they are infinitesimally fragmented.

For the present we do not need to ask how man performs this intervening act—whether with his brain, his stomach, or his glands. Nor do we need to concern ourselves just yet with the essence of the act—whether it is cognitive, conative, or affective. Finally we need not agree on what kind of substance fills the psychological space we have structured—whether the space is stuffed with physiological things, social things, or mental things. All these matters are, at most, no more than subsequent issues, and indeed, as I personally suspect, may prove to be no issues at all, after we have put our mathematics to work.

In this kind of geometrically structured world there are no distances. Each axis of reference represents not a line or a continuum, as in analytic geometry, but one—and only one—distinction. However, there are angles. These are represented by contingencies or overlapping frequencies of incidents. Moreover, these angles of relationship between personal constructs change with the context of incidents to which the constructs are applied. Thus our psychological space is a space without distance, and, as in the case of non-Euclidian geometries, the relationships between directions change with the context.

If we turn from the geometry of the psychology of personal constructs to its arithmetic, we find that the computation is essentially digital rather than analogical, non-parametric rather than parametric. Quantification takes on a different meaning in psychology. But these further implications of our line of theoretical reasoning, exciting as they are, should not be discussed further until after we have talked about more practical matters.

DATA IN TERMS OF PERSONAL CONSTRUCTS

Let us now look at some of the instrumental inventions produced by the psychology of personal constructs, some of the questions the theory poses for psychologists,

some of the methodology for answering these questions, and some of the answers that are beginning to appear.

Probably all of you are familiar with the methods used by Vigotsky to investigate concept formation. One of his methods was to have his subject make a systematic arrangement of small wooden blocks of different sizes, shapes, and colors. The purpose was to see what categories or concepts the subject spontaneously employed, how coherent he was, and whether he could effectively alter his system of categories to meet varying requirements imposed by the experimenter.

Now suppose we consider only the first of these objectives—the observation of what categories the subject spontaneously employs. We shall disregard, for the present, the question of how competent he is or how diligently he complies with the experimenter's whims. Suppose, also, that we are more concerned with the question of how he deals with people than with the question of how he deals with blocks or other inanimate objects. Finally—and this is most important of all—let us look behind the separate categories themselves and focus our attention on the differentiation and integration processes that underlie these categories. In doing all this we shall have digressed widely from what Vygotsky had in mind, though we shall be nonetheless indebted to him for establishing a useful methodological point of departure.

Now, where does this put us? Instead of sorting blocks we shall ask our subject to sort persons he knows. Instead of pointing to the categories into which he places them we shall examine the various ways in which he sees them as different and similar to each other. In other words, instead of cataloging classical concepts, we shall be eliciting psychological data in terms of our basic mathematical unit, the personal construct,

Suppose I give one of you a card and ask you to write on it the name of your mother. Then suppose I give you another and ask for the name of your father. On a third you may write the name of your wife, on a fourth the name of the girl you might have married but did not, on a fifth the name of the professor who influenced you most, and so on until you have a pack of cards containing the names of the most important persons in your life.

Next, suppose I take this pack of cards and select three of them for your particular attention. Perhaps they are the ones on which are written the names of your father,

your former professor, and your present supervisor. I give them to you and ask you to think of some important way in which you regard two of them as similar to each other but in contrast to the third. You look at the cards and then put at one side the two containing the names of your professor and your supervisor, saying, "These two persons have always seemed to know the answers to the questions I asked, but this one here—my father—usually urged me to seek the answers elsewhere."

Now I may give you the card with the name of your brother and ask you where you will place it—with your professor and your supervisor, on the one side, or with your father, on the other. Perhaps you will place him with the father, saying that he, too, was inclined to advise people to find their own answers. Then may come the card with your wife's name on it. How do you apply this differentiation and integration to her? Perhaps, with respect to this particular personal construct of yours, she seems to be more like your professor and your supervisor; that is to say, she thinks she knows the answers to all questions. And so we may go through all the cards in your pack.

The data you have produced may be placed in a simple array with the names of persons arranged in a horizontal row and below them a corresponding row of symbols—plusses and minuses—indicating in each instance whether you regarded each person as more like your professor and supervisor, or more like your father. The personal construct, insofar as it has been explicated by the data you have produced, is now represented in two ways; (1) verbally, by the words you have used to symbolize it, and (2) incidentally, by the row of plusses and minuses. The context in which you have explicated the construct is represented by the particular group of human figures whose names appear on the cards.

What we have done so far may be repeated, starting with another combination of cards. If I give you the names of your wife, your mother, and the girl you nearly married, what outstanding similarity and difference will you see? Perhaps you will say that your wife and mother are sympathetic, but the girl you once thought you loved turned out to be cruel. And what will you say about each of the other persons in this respect? Again, as before, your arrangement of the cards may be recorded as a row of plusses and minuses.

THE FUNCTIONAL IDENTITY THEOREM

We can now examine the two rows of symbols you have produced to see how similar they are. Consider, first, the rather unlikely possibility that the rows are exactly alike, that each person you identified as willing to answer your questions you also judged as sympathetic, and that each person who urged you to find your own answers was also judged as cruel. Consider, further, the limiting case in which these two constructs might be explicated through an infinity of persons and events, but, throughout, would display exactly the same pattern of plusses and minuses. What may we conclude?

If we assume that the rows of plusses and minuses constitute the complete operational definitions of the two constructs involved, we may now conclude that the two constructs are functionally identical, even though you have used different words to describe them. There is an important theorem here to the effect that two constructs which produce an infinite series of identical operations are themselves identical. Of course, in this particular instance, one may raise the objection that since you have used different words in describing the two constructs the operations are not quite infinitely identical. But this is an issue that would force us to turn aside and deal with the whole question of symbols and their peculiar psychological status as events. This is not the occasion for such a digression.

THE CONSTRUCTION MATRIX

We may now pick up our pack of cards again. Starting from various combinations of three cards each, you may produce row after row of comparable data. When we stop we will have displayed before us a rectangular matrix containing a finite number of rows and a finite number of columns of non-parametric entries. At this point we may, if we wish, discard the words you have used to symbolize your constructs and consider only the configuration of incidents in the matrix. We may also discard the names of persons, too; but let us not do that just yet.

It is possible to do many things with this matrix. For example, we may factor-analyze it to see if there are clusters of similar rows or if it may be reduced to two or three row patterns without losing an undue amount of its discriminating power. We can also look at the columns and ask similar questions. What types of people inhabit your world? Are there many types or only a few? And whom do you identify with

whom? Is your wife construed as more like your mother than like any other person you know? Are your father and your supervisor similar in all the respects you have displayed? Do you identify yourself with your father?

Now a host of fresh questions begins to arise, and new issues, both theoretical and practical, emerge. There are questions about the matrix itself. How many of the persons in your life and what sampling of persons is necessary for stable replication of a set of constructs? The indications, so far, suggest that not much is added if more than thirty or forty persons are represented in the matrix. How many constructs should be elicited in order to make the matrix representative of the person's ultimate matrix? This number appears to be smaller; twenty-five or thirty will suffice. The necessary and sufficient matrix is therefore likely to be one that is longer in its horizontal dimension than in its vertical dimension, but we have reason to suspect that these proportions, as well as the required size of the matrix, change as one becomes more mature.

What about changes that take place in the matrix? Do persons undergoing intensive psychotherapy, and who therefore are presumed to be changing radically, see their therapists as more and more like their fathers, as psychoanalytic theory suggests? No; they see them more and more like doctors or like persons who exercise arbitrary authority. Do persons undergoing therapy develop more generalized constructs? To answer this question we must change the subject's sorting task slightly, permitting him to discard those names of persons to whom he cannot apply a given construct. Then we get the interesting answer that in the early stages of psychotherapy the person becomes more restricted in his ways of construing the people he knows. But there is a paradox here. Another study showed that the less restricted he is at the beginning of hospital treatment, the more likely he is to make therapeutic progress—but still that this progress is itself accompanied by cons friction I

But is this what happens in education? No; students during their first year away from home in college seem to change in the opposite way. Their personal constructs become more generalized, particularly constructs which emerge from their new social contacts and experiences.

Genetic Changes in the Matrix

There are questions that may be asked about human development, or the changes that commonly occur as one grows from childhood through adolescence and into adulthood* Not much has been done to answer these questions yet, but there are indications that children in general, as well as certain adults who have failed to adapt themselves to the responsibilities of mature life, tend to rely more upon figures and less upon constructs of the type we have been using for illustration. This is to say the persons they already know are used as direct measures of the persons they meet. For example, when they meet a person for the first time the principal judgment they make is whether that person is totally like or totally unlike their mother. Thus the figure of the mother serves as a kind of concretized personal construct. But as one becomes more mature it appears that his more abstract personal constructs, such as the construct, "sympathetic versus cruel," become the more important reference axes in his psychological space.

There are further changes that seem to come with maturity. In early adolescence one expects to find more use of constructs having an immediate personal reference, such as the "sympathetic versus cruel" construct I have been using as an illustration. But, while such constructs are likely to continue in use through early adulthood, there are others of a more outgoing type that are likely to emerge and assume prior importance. The construct, "willing to answer .questions versus tending to refer questioners elsewhere," is an example of this somewhat more mature construction.

But these changes may happen gradually. In my earlier illustration of the relationship between two constructs I suggested that this latter construct might prove to be operationally similar to the other construct—that sympathetic people were ones who answered one's questions, and cruel people were ones who send you away to find your answers elsewhere. To suggest that you might construe such a relationship was, of course, rather unfair, as I am sure you sensed at the time I mentioned it. It is, instead, the sort of relationship one would most likely find in middle adolescence. Nevertheless, the fact that a person is able to distinguish two such constructs at all suggests that one of them is beginning to separate itself from the other—that these two axes of reference are beginning to rotate away from each other in the person's psychological space. Later on we may even find the relationship somewhat reversed, with a kind of cruelty perceived in those who suppress our

curiosity with too facile answers and a kind of basic sympathy recognized in those who respect the questioner's need to become self-reliant. But it should also be said that by the time this happens both constructs will have been operationally changed, and, indeed, the whole matrix may have been altered considerably.

THE GENERALIZATION OF EXPERIENCE

But now let us turn to another class of questions. If we think of man as we think of a scientist—though we need not think of him as always a "good" scientist—what shall we say happens to his matrix—his theoretical system—when he gets negative results from his experiments. There are several possibilities. He may simply change his particular prediction of what his friends will do, without making other changes in the operations of his personal constructs. Or he may change the grounds for his prediction from one construct to another already present in his repertory. He may also change the operational patterns of the constructs he uses in making his prediction. He may invent new constructs. He may refuse to accept the verdict given by his data and ignore them, distort his perception of them, or manipulate them in such a way that they will appear to confirm his hypothesis. (Incidentally, this latter manipulative reaction provides the basis for a fresh theoretical understanding of hostility—but that is a matter outside the scope of this paper.) Finally, he may change other constructs in his system which, while not used directly in the prediction that has failed, are nonetheless functionally related to those constructs he did use. All these, and others too, are types of changes that can and do occur, but under different conditions.

The limits of this occasion do not allow discussion of all these types of change, but it may be of interest to look more closely into one of them, perhaps the last one I mentioned—changes in constructs not directly used in making the predictions that have failed. For example, if you fail repeatedly when you make predictions of human behavior based on your construct of "sympathetic versus cruel," what happens to that other construct you use—the one that contrasted those who were ready to answer questions with those who turn their questioners elsewhere?

While we are at it, we might as well mention the cognate question; what happens, for example, to your construction of your mother when your wife, whom you have construed to be much like her, turns out to be unpredictable? The first question has

to do with rows in the matrix, and the second with columns. But you will also recognize in the first question the classic issue of response generalization and in the second the classic issue of stimulus generalization, although the matrix provides a different set of parameters for dealing with these problems.

According to the usual notion of a generalization gradient we would be led to suppose that responses that are physiologically much like the one which is changed will be changed also, and that stimuli which are physically much like the one which was misjudged are likely to be reappraised also. Moreover, the gradient falls away from the critical response or stimulus the more unlike it the other responses or stimuli are.

But our matrix enables us to deal with this problem in other terms. Instead of looking to see how physiologically similar to the critical response other responses are, we can look at the matrix and ask how psychologically similar they are. And the same is true in the case of stimuli; instead of taking account only of their physical similarity, we can examine their functional similarity. Thus we do not ask how much physiological similarity there is between one's responding to persons who appear sympathetic and responding to those who answer questions; we ask, instead, if the conditions for eliciting the responses are the same. And on the stimulus side, we do not ask if your mother and your wife are actually alike; we ask, instead, if your responses to them are usually the same.

Now, with the problem of generalization set up in the psychomathematical terms of the matrix instead of the physiological and physical terms we have customarily used, what do we find? The results, so far, are not conclusive but they strongly suggest that the popular notion of the generalization gradient does not hold when reduced to these terms. If your construct of "sympathetic versus cruel" fails you and you change it, and if your construct of "answering versus sending you after your own answers" is functionally similar—though not too similar—you may, instead of making a moderate change in your use of the latter construct, become quite rigid. Similarly, if your wife and your mother are construed in much the same fashion, your disillusionment with your wife is likely to be accompanied by an idealization of your mother. Your wife, so important to you, must be reconstrued in many ways, so your mother, also important to you, you are unwilling to reappraise in any way whatsoever.

Suppose we look at the reverse situation. What happens when one's predictions are consistently confirmed? Still dealing in the same parameters and with the same tentativeness about conclusions, we find our gradients inverted. It seems, then, that at those points where one is certain of his outcomes he holds fast to his constructions but becomes freer to explore variations in adjacent areas. If you are sure of your wife you can take chances with your mother, or if you are confident of your mother you can be flexibly responsive to changes in your wife—though this is predicated on your original construction of them as similar to each other. Conversely, if you are sure about the meaning of sympathy and cruelty, you can take a second and more mature look at what it means to give glib answers to questions—though this, too, is predicated on an initially close functional relationship between these two constructs.

Tentative as they are, these still are conclusions which gain support from incidental sources. Clinical experience suggests that the child who has a reliable understanding of what is happening at home is the first to venture into the next street where he suspects strange things are going on. And we have seen that children who have found their homes chaotic may develop irrational and inflexible attitudes toward those who function as parents. Or in psychotherapy: There it often proves helpful for the therapist to establish himself as a reliable father-like figure before that timid fellow-scientist of his—the patient, dares reexamine and experiment with his parental relations.

There are other parameters, derivable from the matrix, which bear on this problem of generalization, but perhaps I have said enough to indicate that the matrix is a fertile ground for exploration and that the mathematical operations it supports can be put to good use.

THE CONSTRUCTION MATRIX AS A GENERAL MATHEMATICAL FORM

Now I would like to correct a false impression I may have allowed to occur. I have talked about the matrix as if it were a particular psychological instrument, with persons ranged along one axis and constructs ranged along the other. But this is only a particular case. We could have substituted occupation **or** job assignments instead of persons and, as one investigator has done, examine the resulting matrix to see how workers construe their tasks and why they do some of the things they do.

Or, if we are dealing with children, we could use the matrix to explicate the relationship between toys and games. Or, if we are dealing with rats, we could set up conditioning series to different triads of signals, and then examine the matrix to see what relationships emerge, and the effects of extinction on the patterns of relationship.

Finally, as I think of the uses to which the matrix might be put, I find myself a little depressed. Suppose someone would surreptitiously put "stimuli" instead of persons along one margin of the matrix, and "responses" instead of constructs along the other. If that should ever happen I am sure I would feel that I had been brought back, full circle to where I started. But perhaps no one will be so unkind as to do this; perhaps the most that will happen is that someone will put Leningrad "signals" along one axis and Tbilisi "sets" along the other. In any case, let us say broadly that the matrix is a general mathematical operation for relating events and behavior, and that the concurrence of these two psychological values can be expressed in terms of the psychomathematical function I have described—the personal construct.